Applied Real Options Analysis

For the Finance and Decision Professional

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Outline

- *Project Management* minimizes risks via managerial flexibility
- *Traditional Valuation* does not include the value of flexibility
- Integrate project management with financial valuation

- Valuation
  - Pure Valuation - reporting
  - Investment Decisions
- Optimal Controls and Policies
  - Determine optimal operating policies

What are Real Options

Definition (Eduardo S. Schwartz - UCLA)

Real options are contingent decisions that provide the opportunity to make a decision after uncertainty unfolds.

- Future actions in response to new information drive option value
- Firms can be thought of as bundles of real options
- ROA captures how businesses actually operate

Some Examples

- Options to Expand
  - Netflix starting a movie studio
- Option to Switch
  - Ethanol Plants at the start of the pandemic in 2020 mothballed plants or switched to producing hand sanitizer.
- Option to Learn
  - R&D in biotech, pharmaceuticals, semi-conductors, etc.
- Option to wait
  - Delay new store openings

Traditional Valuation Approach - DCF

Recall:

$$Static\ NPV = \sum_{t=0}^{T} \rho^t C_t$$

- C_t is the cash flow in period t
- $\rho^t = \frac{1}{(1+r)^t}$ is the discount factor
- $T < \infty$ is the finite terminal time period
- Static NPV does *not* depend on actions

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- s_t is the state of the project in time period t
- $C_t(s_t, a_t)$ is the cash flow in period t and is now a function of the current state and the action taken

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- $Dynamic\ NPV = 150 - 0 + 175 = 325$

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- Optionality: $\max\{C_t, 0\}$
- $static\ NPV = 150 - 100 + 175 = 225$
- $Dynamic\ NPV = 150 - 0 + 175 = 325$
- $Option\ Value = 325 - 225 = 100$

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  - Path dependent constraints

How to Solve Real Option Models

- Binomial Trees/Lattices
- Partial Differential Equations - Finite Difference Methods
- Regression Monte Carlo

How to Structure American Options

Recall the sequence Problem:

$$V_0(s_0) = \max_{\{a_t\} \in \mathcal{A}} \sum_{t=0}^{T} \rho^t C_t(s_t, a_t)$$

Transform it into a recursive problem:

$$V_0(s_0) = C_0(s_0, a_0) + \sum_{t=1}^{T} \rho^t C_t(s_t, a_t)$$

$$V_0(s_0) = C_0(s_0, a_0) + \rho V_1(s_1)$$

Generalize previous slide and add uncertainty:

$$V_t(s_t) = \max_{a_t \in A(s_t)} \left(C_t(s_t, a_t) + \rho \mathbb{E}[V_{t+1}(s_{t+1}) | s_t]\right)$$

- $C(s_t, a_t)$ is current cash flow or contribution
- $s_{t+1} = f(s_t, a_t)$ is the state transition function
- $\mathbb{E}[V_{t+1}(s_{t+1}) | s_t]$ is the *continuation value*
- $\mathbb{E}[V_{t+1}(s_{t+1}) | s_t] = ?$

Estimate Continuation Values

Use Ordinary Least Squares:

$$\mathbb{E}[V_{t+1}(s_{t+1})|s_t] \approx \sum_{i=1}^{m} \beta_i \phi_i(s_t)$$

- ϕ_i are the basis functions
- β_i are the regression coefficients
- OLS is one option of many for approximation

States

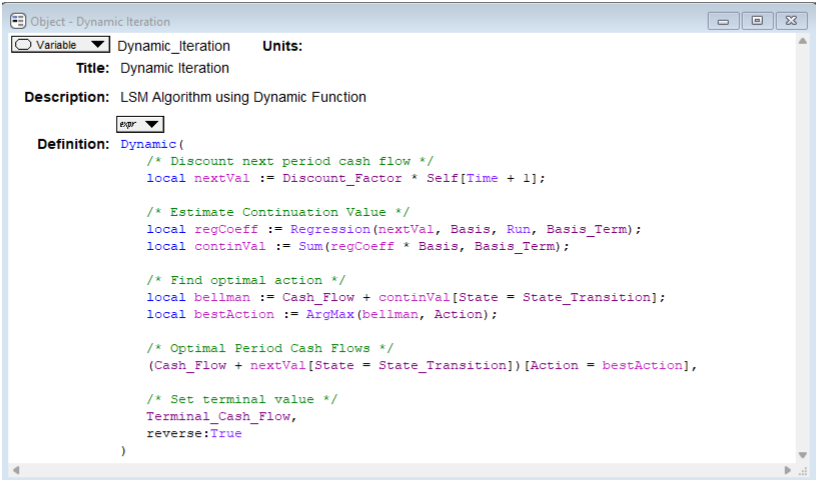
States can be decomposed into two types:

1. Endogenous States
    - States that the action or decision will influence
    - E.g. modes of operations (Operating, suspended, mothballed, abandoned)
2. Exogenous States
    - The component of the state that is not influenced directly by the decision
    - E.g. prices and costs
    - Usually the stochastic variables
    - Uncertainty plays a critical role

LSM Algorithm

1. Define states, actions, basis functions
2. Simulate the stochastic states
3. Calculate the Terminal Cash Flow and set as initial estimate of the project value
4. Iterate from $T-1$ to 1:
    1. Discount Next Period's Project Value
    2. Regress discounted PV against current period's basis
    3. Estimate Continuation Values
    4. Form Bellman Equation
    5. Determine Optimal Action
    6. Update Project Value Estimate
5. Discount PV and average across all runs to determine period 1's Continuation Value
6. Solve period 0's Bellman Equation

Analytica LSM Algorithm




```
Dynamic(
    /* Discount next period cash flow */
    local nextVal := Discount_Factor * Self[Time + 1];

    /* Estimate Continuation Value */
    local regCoeff := Regression(nextVal, Basis, Run, Basis_Term);
    local continVal := Sum(regCoeff * Basis, Basis_Term);

    /* Find optimal action */
    local bellman := Cash_Flow + continVal[State = State_Transition];
    local bestAction := ArgMax(bellman, Action);

    /* Optimal Period Cash Flows */
    (Cash_Flow + nextVal[State = State_Transition])[Action = bestAction],

    /* Set terminal value */
    Terminal_Cash_Flow,
    reverse:True
)
```